The Eleatic and the Indispensabilist

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January 27, 2014

7942 words

Abstract:

The debate over whether we should believe that mathematical objects exist quickly leads to the question of a criterion for determining what we should believe to exist. Indispensabilists claim that we should believe in the existence of mathematical objects because of their ineliminable role in scientific theory. Eleatics argue that only objects with causal properties exist. Mark Colyvan's recent defenses of Quine's indispensability argument and its criterion, against some contemporary eleatics, presents an intriguing attempt to provide reasons to favor the indispensabilist's criterion. I show that Colyvan's argument is not decisive against the eleatic and then sketch a way to capture some of the important intuitions behind both views.

§1: Introduction

The debate over mathematical realism is old and open.[1] W.V. Quine argued that we should believe in the existence of mathematical objects because of their indispensable uses in scientific theory. In contrast, philosophers with various motivations have argued for an eleatic principle which states, roughly, that only causally-active entities exist. The eleatic principle is sometimes invoked by its proponents to deny that we should believe in the existence of mathematical objects. So the debate over whether we should believe that mathematical objects exist can quickly lead to the question of how we should determine what to believe exists.

One danger of such a debate is that mathematical platonists, those who believe in the existence of mathematical objects, tend to prefer criteria which allow for the existence of mathematical objects. In contrast, mathematical nominalists, those who deny the existence of mathematical objects, tend to favor restrictive criteria which eliminate the possibility of justifying beliefs in mathematical objects. The trick is to formulate criteria which do not beg the question.

Mark Colyvan defends Quine's indispensability argument and its criterion for determining our ontological commitments against contemporary eleatics. One thread of Colyvan's argument is promising precisely because it may be seen as independent of the debate over mathematical objects. Colyvan provides examples which could show that the eleatic principle is unacceptable even when applied to the non-mathematical portions of scientific theory.

In this paper, I first introduce the indispensabilist and eleatic positions, framing the debate so that it avoids being question-begging. Then, I show that Colyvan's argument is not decisive against the

[1] Mathematical realism, or platonism, is sometimes taken as the claim that some mathematical propositions are non-vacuously true, called sentence realism, and sometimes as the claim that some mathematical objects exist, called object realism. I'll work here with object realism which is the same as sentence realism on the assumption of a standard semantics for mathematical sentences (i.e. one for which some mathematical sentences are non-vacuously true if and only if some mathematical objects exist).

eleatic. While the form of Colyvan's argument is correct and could be used to show the inapplicability of the eleatic principle, the examples themselves are unsuccessful. In the end, I sketch a platonist view concerning mathematical objects which captures some of the advantages of each side. We can adopt both the eleatic's claim that causal efficacy is central to our beliefs about how to understand the ontology of scientific theory and the indispensabilist's claim that some mathematical propositions are true.

§2. Quine's Argument

Quine's indispensability argument proceeds as follows.[2]

QI
- QI1. We should believe the theory which best accounts for our sense experience.
- QI2. If we believe a theory, we must believe in its ontological commitments.
- QI3. The ontological commitments of any theory are the objects over which that theory first-order quantifies.
- QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
- QIC. We should believe that mathematical objects exist.

QI follows from Quine's general method for determining the ontological commitments of a theory. First, QI1 - QI2, we choose a best theory. Then, QI3, we regiment that theory in a canonical language of first-order logic with identity. Last, QI4, we examine the domain of quantification of the theory to see what objects the theory requires in order to come out as true.

Quine's method applies to any theory. Theories which refer to trees, electrons, and numbers, and theories which refer to ghosts, caloric, and God, are equally amenable of Quine's general procedure. The theory to which QI refers, and which purportedly yields mathematical objects, is designed to explain our sense experiences. In addition to its references to ordinary objects, like trees, it refers to some objects which we can not, arguably, sense directly, like carbon atoms, and to some objects which we can not

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[2] Quine nowhere presents a detailed indispensability argument, though he alludes to the argument throughout much of his work. See Quines 1939, 1948, 1951, 1955, 1958, 1960, 1978, and 1986. Quine and Goodman 1947 is a notable exception. QI is my reconstruction of Quine's argument.

sense at all, like sets. Some of the objects posited by my theory are constituents of ordinary objects, as carbon atoms constitute portions of trees. Other objects to which my theory refers (e.g. mathematical objects) are posited for more formal or technical reasons.

An often-overlooked virtue of Quine's argument, and one worth keeping in mind, is that it obviates any worries about access to mathematical objects.[3] Our ontology is not determined by any reductionist account of our sense experience. We need not see mathematical objects in order to believe that they exist. Rather, our ontology is determined by the construction and interpretation of first-order versions of our theories.[4]

There are a variety of possible negative responses to the Quinean argument. One possibility is to deny QI4.[5] Another possibility is to deny Quine's method for determining the commitments of a theory.

§3: The Eleatic Principle

Eleatics like David Armstrong defend an alternative to Quine's method. The eleatic principle, which has its roots in Plato's *Sophist*,[6] is notoriously difficult to articulate precisely. This difficulty arises in part because necessary and sufficient conditions are always hard to formulate. Armstrong

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[3] See MacBride 2004 and [author's work].

[4] Quine insists that our ontological commitments are best expressed in theories written in the language of first-order logic with identity. Some scientific and mathematical theories may be more attractive when regimented in another (e.g higher-order) language. Moreover, some important theories admit of no first-order axiomatization; see Luce et al. 2007, Chapter 21. There are deep and interesting questions about how to resolve this tension within a Quinean framework.

[5] Field 1980 spurred a range of projects denying QI4 by rewriting either mathematical or scientific theory to dispense with quantification over mathematical objects. Burgess and Rosen 1997 elegantly compiles many of these dispensabilist strategies. We can easily eliminate quantification over mathematical objects from first-order theories of standard science, using a variety of tricks which yield ugly, unwieldy theories. So, the Quinean must emphasize the 'best' in both QI1 and QI4. For the purposes of this paper, I grant QI4 and set aside the questions of whether dispensabilist projects succeed.

[6] "I am proposing as a mark to distinguish real things that they are nothing but power" (*Sophist* 247e, Cornford translation).

emphasizes both causal activity and spatio-temporal location:

> Against the suggestion that the world might contain...such things as possibilities, timeless propositions and "abstract" classes, I argued that these latter entities had no causal power; and that if they had no power there was no good reason to postulate them (Armstrong 1978b: 46).
>
> The world is nothing but a single spatio-temporal system (Armstrong 1978a: 1:126).

In response, Oddie 1982 develops counter-examples to these and other expressions of the principle. The difficulty in formulating the principle precisely is compounded by its reliance on a concept, causation, which is itself notoriously unclear; the concept of causation has befuddled philosophers since at least Hume's time, and remains obscure.

Despite difficulties formulating a precise eleatic principle, it is not unclear how the principle is intended as an alternative to QI regarding whether we should believe in mathematical objects. QI says that we should believe in mathematical objects since they are ineliminable elements of our best theory. The eleatic claims that we should not believe in mathematical objects, even if they are included in our best theory, because they are not causally connected to the directly sensible, space-time world. They are mere heuristic devices. "If any entities outside the [spatio-temporal] system are postulated, but have no effect on the system, there is no compelling reason to postulate them" (Armstrong 1980: 154).

Let's put aside worries about specifying the eleatic principle precisely since the following paradigmatic formulation will suffice here:

EP Only those things which are causally active are real.[7]

The eleatic need not deny the indispensabilist's claims, at QI1 and QI2, that we should believe our best scientific theory and the posits that it makes. S/he need not deny the naturalism or the holism that underlie those premises. Nor need the eleatic deny QI4, the claim that mathematics is ineliminable

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[7] Compare to Oddie 1982: 286; Azzouni 2004b: 150; and Field 1989: 68.

from our best theory. The central disagreement between the eleatic and the indispensabilist concerns how we read the posits from the theory, QI3. The eleatic claims that to be is to be causally active; our theory contains both real posits (e.g. trees, carbon atoms) and instrumental ones (e.g. the square root of two). The real posits exist and the instrumental ones do not. The indispensabilist claims that to be is to be the value of a variable, that to call a posit a posit is not to patronize it, and that any distinction between real and instrumental posits in our best theory is arbitrary.

In contrast to the indispensabilist's latter criticism, the eleatic believes that s/he has a principled, rather than arbitrary, way of distinguishing between real and instrumental elements of a theory: on any reasonably good theory, mathematical objects are causally isolated from the rest of the world. Mark Balaguer calls the fact that we are unable to interact with mathematical objects the principle of causal isolation, or PCI. The commonsensical PCI is an eleatic principle and Balaguer wields it against the indispensability argument, noting how QI and EP are directly in tension. "The Quine-Putnam argument should be construed as an argument not for platonism or the truth of mathematics but, rather, for the *falsity of PCI*" (Balaguer 1998: 110).

The indispensabilist, embracing mathematical objects, accepts the existence of causally isolated entities. The eleatic, unwilling to accept anything causally isolated, rejects mathematical objects. A recent eleatic, Joseph Melia, argues that we can differentiate among the posits of scientific theories which quantify over mathematical objects according to their causal roles.[8]

> The mathematics is there to enable us to express possibilities that may be otherwise inexpressible but it plays no real role in simplifying our picture of the world... Postulating quarks genuinely makes the *world* a simpler place. Under the quark hypothesis, various objects in the particle zoo do exist *in virtue of* the existence, properties and relations of quarks (Melia 2000: 474).

[8] Melia does not explicitly formulate his claim as EP, but it does the same work. Azzouni 2012 characterizes Melia's strategy as evading QI3 rather than denying it; I don't think that Azzouni's distinction matters here.

Similarly, Mary Leng's argument for taking mathematics as merely recreational relies on a claim that mathematics is insulated from the rest of science.[9] Its role is merely descriptive, for purposes of modeling physical phenomena. "When we use mathematics in science we do not invoke the existence of mathematical objects. Rather, we interpret our strictly meaningless mathematical terms by tying them to scientific phenomena, and use their mathematical consequences to draw conclusions about the scientific phenomena" (Leng 2002: 411).

In sum, the mathematical platonist may find QI convincing. The anti-platonist may be convinced by EP. The indispensabilist's appeals to quantification over mathematical entities fail to convince the eleatic to abandon her/his instrumentalism about them. The eleatic's appeals to the causal inefficacy of mathematical posits fail to appear relevant to the indispensabilist. Stalemate.

To decide between the two competing positions without begging the question (of the existence of mathematical objects), we need reasons independent of mathematics to adopt one or another of the competing principles QI3 or EP. In the next three sections I will show that recent attempts to provide such reasons, on both sides, do not succeed.

§4: Defending EP

Jodi Azzouni defends EP by arguing that adopting QI commits us to objects we do not really believe exist. He describes instances in which existential quantifications within science proper should be seen as merely instrumental. The users of scientific theories are not committed to centers of mass, quasi-particles, and mathematical objects. While the latter example is question-begging for the purposes of this paper, the first two examples are worth a moment's reflection.

Azzouni 1997b considers a system of two masses connected by a spring, moving in a

[9] Leng extends the use of 'recreational' found in Quine 1986: 400, where Quine uses it to refer to portions of mathematics not applied in scientific theory.

gravitational field. The separate motions of the masses are too complicated to calculate. But we can describe the system if we consider it in terms of its center of mass, which is not located on the springs, and its reduced mass. Our description of the system refers to its center of mass indispensably, according to Azzouni. Yet we know that the center of mass is not a real thing. It is a merely instrumental posit. Thus Quine's method does not yield the proper results.

Turning to the second example from Azzouni's work, quasi-particles are posits used to replace one intractable many-body problem in condensed-matter physics with many one-body problems, using Fermi Liquid theory. Scientists introduce quasi-particles aware that a fictionalization is involved. "[I]t's not that physicists are failing to ask whether or not they're committed to the entities introduced in this way. *They already take themselves* not to be so committed. That's why, for example, such 'particles' are called quasi-particles" (Azzouni 1997b: 195).

Azzouni urges us to cleave ontological commitment from the existential quantifier while maintaining the quantifier's inferential role. We may leave our theories written as they are. But we should not read existential quantifications as indicating what exists. If we want to reveal our commitments within formal scientific theory, we can introduce a predicate to be read as 'is physically real'.[10] The principle which would guide Azzouni's ascriptions of such a predicate is that we have thick epistemic access to anything physically real. Observation is paradigmatically thick access. "Any form of epistemic access which is robust, can be refined, enables us to track the object..., and which (certain) properties of the object itself play a role in how we come to know (possibly other) properties of the object is a *thick* form of epistemic access" (Azzouni 1997a: 477).

Azzouni's defense of EP thus relies on an epistemic argument: we have no reason to believe in

[10] See Azzouni 2004a: 383; and much of Azzouni 2004b, especially Chapter 4.

causally idle entities because we lack thick or even thin access to them.[11] We have thin access to objects to which we lack thick access but to which we could, in principle, have that access. If we have thin access to an object, we can provide an explanation (or excuse clause) of why we lack thick access to it. For example, we can have thin access to objects outside our light cone, an asteroid, say. Despite our lack of access to such an asteroid, we can understand that it can be as real as those in the belt between Mars and Jupiter.

Colyvan, in earlier work, argues that epistemic arguments for EP, like Azzouni's, are anthropocentric.[12] The charge of anthropocentrism against Azzouni is mitigated, to some degree, by his acceptance of thin posits; we need not observe something in order to believe that it exists, but we need to have some (presumably causal) story about why we lack access: it's too small, or it's too far away. More recently, Colyvan seems to take Azzouni's epistemic argument to be more promising. "The beauty of Azzouni's thick and thin epistemic access approach is that it does not seem to beg the question against platonism and yet, according to Azzouni, it does rule against ontological commitment to abstract entities such as numbers" (Colyvan 2010: 5).

Colyvan is correct that we want a way to determine whether mathematical objects exist that avoids begging the question. It will not suffice to deny that we should believe that mathematical objects exist because we lack access to them; the question only arises because of our lack of access. To know if we should take our commitments to mathematical objects seriously, we require some independent criterion. "Whether mathematical objects are thin or very thin depends on what can count as an excuse for not being accessed thickly... Unfortunately Azzouni doesn't give us any guidance; he offers no systematic story about acceptable excuse clauses. Moreover, the excuse clauses play a central role in

[11] Azzouni actually presents four levels of epistemic access: thick, thin, very thin, and ultra thin. See Azzouni 2004: Chapter 6.

[12] See Colyvan 2001: §3.2 and Colyvan 1998: §3. See also Colyvan 2010: fn 10, in which he endorses his earlier argument.

Azzouni's account, so independently of concerns about mathematical entities, a well-motivated and detailed account of what passes for an excuse is required" (Colyvan 2010: 7).

In order to accept a non-sensible posit as a real thing, Azzouni requires an explanation of our lack of access to it. Thus, while it may appear that Azzouni's distinctions among posits avoids begging the question against the platonist, they are really just refinements of the underlying reasons for those distinctions: EP.[13]

To an end similar to that of Azzouni, Penelope Maddy cites skepticism surrounding atoms in the early stages of atomic theory.[14] Before the experiments were conducted which yielded more direct evidence of the existence of atoms, many scientists were skeptical of these elements. Atomic theory was accepted. It could be naturally taken as quantifying over atoms. But at least some scientists did not really believe that atoms existed.[15] Maddy cites other examples of false assumptions in science: taking water waves to be infinitely deep, and treating matter as continuous in fluid dynamics. According to Maddy, it is accepted scientific practice to separate our actual commitments from those made by our best theories. "If we remain true to our naturalistic principles, we must allow a distinction to be drawn between parts of a theory that are true and parts that are merely useful. We must even allow that the merely useful parts might in fact be indispensable" (Maddy 1992: 281).

One might argue that the views of scientists in cases such as Azzouni's example of quasi-

[13] "Azzouni's main motivation in his earlier articles was the idea that mathematical entities are causally idle and therefore idle *simpliciter*. As I have already mentioned, I think that this idea is still prominent in his current thinking" (Colyvan 2010: 7). Azzouni denies that he needs to formulate excuse clauses. "[I]t's not my job to give a well-motivated and detailed account of what passes for an excuse. Such excuses issue from science: my job is only to note that fact and indicate its importance (which I've done)" (Azzouni 2012: 962).

[14] Maddy aims to withhold truth from sentences of the theory, while Azzouni wants to avoid commitments to entities. This difference does not matter here.

[15] See also Maddy 2005: §II.3 on how we might (perhaps with Einstein) withhold belief in the continuum despite accepting general relativity theory.

particles or Maddy's example of atomic theory are irrelevant to the defender of QI. QI is only interested in the commitments of our best theories, not in the commitments of the scientists who present or evaluate those theories. But the question of which theory is best is difficult to answer. Scientists do not use any canonical form in practice. There are different ways to regiment any scientific theory. The views of scientists about how to understand their theories are thus relevant to the question of the commitments of those theories. An atomic skeptic might prefer a Ramsified version of atomic theory, for example, where a realist might prefer one which quantifies over atoms.

Maddy and Azzouni have given us examples in which quantification over a class of entities (centers of mass, quasi-particles, atoms) does not track our beliefs about their existence. I do not take these cases to be decisive in favor of EP, as I will explain in §6. Still, they are the kinds of examples we are seeking in that they attempt to avoid begging the question of the existence of mathematical objects. Let's turn to Colyvan's parallel cases in favor of QI.

§5: Colyvan's Defense of QI

Colyvan defends the indispensability argument, and Quine's method, against the eleatic. He argues that we are committed by physical theory to non-causal entities which play indispensable explanatory roles.[16] If we admit non-causal, non-mathematical objects, then the eleatic principle fails independently of what we think about mathematical objects. The door would be open to admit mathematical objects as well. And, Colyvan argues, there are good reasons to admit non-causal non-mathematical objects. Thus, according to Colyvan, the principled distinction which supports EP is wrong.

[16] Indispensability arguments must present some goal for which commitment is indispensable. For Quine and QI, this goal was the construction of scientific theory. Colyvan focuses on scientific explanation, as Armstrong did. The examples play the same role in both domains.

Colyvan presents three examples.[17] In the first example, Colyvan argues that the best explanation of light bending around large objects is geometric, rather than causal. "It's not that something *causes* the light to deviate from its usual path; it's simply that light travels along space-time geodesics and that the curvature of space-time is greater around massive objects" (Colyvan 2001: 47-8). Large masses covary with curvatures in space-time, but it is not clear, on a causal picture, which causes which. Furthermore, according to the non-Minkowski vacuum solutions to the Einstein equation, there are empty, yet curved space-times. On the causal picture, these curvatures are uncaused, and thus unexplained.

Colyvan's second example concerns the existence of two antipodes in the Earth's atmosphere with exactly the same pressure and temperature at the same time. The causal explanation of this phenomenon, which refers to atmospheric conditions, suffices only to describe the existence of the antipodes, and does not explain why they inevitably exist. The existence of antipodes is guaranteed by a topological theorem. The proof of this theorem provides the remainder of the explanation and is non-causal.

Third, Colyvan asks us to consider the Fitzgerald-Lorentz contraction. A body in motion contracts, relative to an inertial reference frame, in the direction of motion. Minkowski's explanation of this contraction relies on equations in four dimensions, representing the space-time manifold. Colyvan calls this, "A purely geometric explanation of the contraction, featuring such non-causal entities as the Minkowski metric and other geometric properties of Minkowski space" (Colyvan 2001: 51).

To evaluate Colyvan's examples, recall that we are looking for non-causal entities other than mathematical objects which play an explanatory role. His argument is that since we need non-causal

[17] Two comments: First, the three examples are present in Colyvan 2001, but only the second and third are present in Colyvan 1998 in which the problem is framed more aptly for present purposes. More importantly, the role of these examples in Colyvan's work is not always explicitly framed. In defending QI, Colyvan is fighting a two-front war: against the eleatics who deny QI3 and against the dispensabilists who deny QI4. Colyvan's examples play roles on both fronts, supporting QI4 against dispensabilists like Hartry Field as well as opposing EP. My criticisms of Colyvan's examples in this paper apply only to their use against the eleatic and not to their use supporting QI4 against the indispensabilist.

non-mathematical elements in our best theory, EP is shown false independently of the contentious mathematical case. If Colyvan's examples were to show only that mathematical elements of our best theory were indispensable, then using those examples to choose QI over EP would be just like using a bare preference for mathematical realism to choose QI over EP or a bare disdain for mathematical objects to choose EP over QI.

Colyvan's first example, the geodesics, either begs the question or is insufficient. If we take the geodesics as pure mathematical objects, Colyvan begs the question by presenting a geometric object as explanatory. If we take geodesics to be physical entities, then we should see them as properties of physical space-time, as opposed to objects of pure geometry. Ascribing causal properties to mathematical entities may be repugnant, but taking physical geometry to have causal relations to physical objects is not. We may naturally see masses as causing curvatures in physical space.

Colyvan rejects the causal interpretation. "[A]ny account that permits mass to *cause* the curvature of space-time is unintuitive to say the least" (Colyvan 2001: 48). The unintuitiveness, for Colyvan, may arise from thinking of space-time as abstract, or relationalist. If we think of it substantivally, the causal explanation is not problematic. Indeed, any non-causal explanation of the curving geodesics near massive objects would make those curves seem entirely accidental. Without massive objects in or near their paths, light rays travel in straight lines.

The case of an empty, yet curved, space-time only reinforces the claim that we do not need a non-causal, non-mathematical explanation. The curvature of space-time is not an event which can be explained in terms of antecedent conditions, say. We can take it, with the substantivalist, to be a property of an object or collection of objects: space-time points or regions. Or, we can take an empty yet curved manifold as a pure geometric object. In neither case do we need to posit non-causal, non-mathematical objects.

In Colyvan's second case, the antipodes, we must again make a pure/applied distinction regarding the topological theorem. The pure mathematical theorem does not guarantee that these antipodes have the same temperature and pressure. We need bridge principles which apply this theorem to the Earth and its weather patterns. Once we add these bridge principles, the proof which guarantees the antipodes may naturally be regarded as a causal explanation. For, the bridge principles will refer to causal structures within the Earth's atmosphere, and it is these which explain the existence of the antipodes. This explanation will, as Colyvan notes, refer to non-causal entities such as continuous functions and spheres, but these are mathematical objects. We are looking for non-causal, yet non-mathematical, elements.

In the third example, the equations which explain contraction are supposed to make indispensable reference to non-causal entities: the Minkowski metric and other geometric properties of Minkowski space. Geometry, as in the bending-of-light example, can either be taken as a purely mathematical theory or as a description of the space-time manifold. In the latter case, we again may take space-time substantivally, in which case the geometric explanation of the contraction need not appeal to non-causal elements.[18] In the former case, the Lorentz contraction is explained by a combination of mathematical objects (the equations which describe the transformations) and physical objects (the objects contracted). The equations apply to the physical world, and thus explain the contraction of a physical body in motion, when coupled with bridge principles which explain their applicability.[19]

It is a perennially interesting question why mathematical objects are applicable to physical theories, as in these cases. But, the indispensability of mathematics may be granted by eleatics without weakening their argument against the indispensabilists at all. Melia and Leng claim that scientists use mathematics in order to express facts that are not representable without mathematics but that such

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[18] For a defense of space-time substantivalism, see Field 1989, Chapter 6.

[19] Melia writes similarly: "The Minkowski explanation is a *geometric* explanation of relativistic effects - not a mathematical one" (Melia 2002: 76).

representations are not supposed to be ontologically serious. “The mathematics is the necessary scaffolding upon which the bridge must be built. But once the bridge has been built, the scaffolding can be removed” (Melia 2000: 469). References to mathematical objects in our best scientific theories need not impel us to believe in them. “We are not committed to belief in the existence of objects posited by our scientific theories *if their role in those theories is merely to represent configurations of physical objects*. Fictional objects can represent just as well as real objects can” (Leng 2005: 179).

In none of his three cases has Colyvan shown that a non-causal entity, other than a mathematical object, plays an essential role in scientific explanation. The eleatic, *ex hypothesi*, need not show that mathematical entities can be removed from explanations in the physical world. Thus, Colyvan’s argument does not show that QI should be preferred to EP.

Furthermore, it is difficult to see how any such examples could convince the eleatic to cede EP in favor of QI. The strategies I used in this section (e.g. appealing to bridge laws, distinguishing between physical and pure geometry) are always available to the eleatic no matter what examples the indispensabilist proffers.[20]

§6: Ideal Theories and Double-Talk

Given the failure of Colyvan’s examples to establish QI over EP, we might decide in favor of EP. Whether we do may seem to depend on some precedental examples like those I examined in §4. In those examples, the eleatic denies the existence of an object over which a good physical theory may quantify: 1. Centers of mass; 2. Quasi-particles; and 3. Atoms, prior to experimental verification. I now wish to show that such examples do not establish EP over QI either.

It is fairly easy to see how a defender of QI could deny that we should believe in the existence of centers of mass and quasi-particles. They are introduced as idealizing fictions, to make intractable

[20] For example, Bangu 2008 responds to Baker 2005 similarly.

calculations possible. Engineers and experimental physicists require such idealizations. But, says the defender of QI, our best theory is not the one we use for practical purposes. Our best theory will be the one in which we regiment our most sincere, austere commitments. Similar remarks can be made against Maddy's examples of false assumptions within science, like taking water waves to be infinitely deep. Any theory which takes the Caribbean Sea to be infinitely deep is clearly not a theory we should use to reveal our ontological commitments. Maddy's case of the atoms is a bit trickier. Recent scholarship argues that Mach, who is often cited as one of the atom skeptics, was not as skeptical as he is ordinarily portrayed.[21] Still, Maddy's claim that we often remain skeptical of the referents of even our best theory does not depend on this one case. The Quinean will accuse any eleatic, including the atom skeptic, of intellectual dishonesty: you may not deny that which your theory demands.

This double-talk criticism is essential to QI: if our best theory requires electrons for its bound variables, then we should believe in electrons; if it requires sets, then we are committed to sets. We can not assert the existence of objects at one moment and then take back those assertions at the next, on pain of inconsistency.[22]

The double-talk criticism appears throughout Quine's work. Quine's response to Carnap's internal/external distinction relies on it. Once one has accepted mathematical objects as an internal matter, one can not merely dismiss these commitments as the arbitrary, conventional adoption of mathematical language. Quine's response to the Meinongian Wyman in "On What There Is," is also a double-talk criticism. Wyman presents two species of existence, but Quine distinguishes between the meaningfulness of 'Pegasus' and its reference in order to avoid admitting that Pegasus subsists while at the same time denying that Pegasus exists. Putnam, defending Quine's indispensability argument, makes

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[21] See Banks 2004.

[22] Not everyone believes that all such double-talk is illicit. See Melia 2000. Insofar as one accepts double-talk, one must reject QI.

the double-talk criticism explicitly. “It is silly to agree that a reason for believing that p warrants accepting p in all scientific circumstances, and then to add ‘but even so it is not *good enough*’” (Putnam 1971: 356).

Worries about double-talk bother Quine’s critics as well as his supporters. Field applies the double-talk criticism directly to worries about mathematics. “If one *just* advocates fictionalism about a portion of mathematics, without showing how that part of mathematics is dispensable in applications, then one is engaging in intellectual doublethink...” (Field 1980: 2).

The double-talk criticism of the eleatic by the indispensabilist is, rightly, a non-question-begging methodological principle. The two parties seem to be disagreeing about bare questions of ontological commitment: do mathematical objects exist or not? This object-level debate leads to a deeper meta-philosophical disagreement: which method of determining our ontological commitments is correct? To use the methodological principles to support one’s view about the existence of mathematical objects, one must find a way to defend those methodological principles without appealing to the mathematical case. The double-talk criticism of EP does that.

Unfortunately, while Quine’s method seems to avoid the eleatic’s double-talk of denying that mathematical objects exist while using them ineliminably in scientific theory, indispensabilists are liable to commit their own sort of double-talk. If the defender of QI responds to examples like those of Azzouni and Maddy by retreating to an ideal best theory, s/he may also be speaking equivocally: we use centers of mass (quasi-particles, atoms), but we don’t really believe that they exist.

This point, that the Quinean appeal to ideal theories masks a kind of double-talk, should not be underestimated. Criteria for theory choice are notoriously complicated even if QI would be stronger if they were categorical and objective. We balance elegance, parsimony, breadth, unification, and simplicity, among other characteristics. There are no precise formulas for how to weigh these complex, interactive factors. We are free to choose among various ways to present a theory: various canonical languages, various mathematical axiomatizations, various formulations of the empirical axioms. For example, Maxwell's equations for electromagnetism can be presented in integral or differential formulations, using a Cartesian basis, vectors, tensors, or even quarternions,[23] using SI or Gaussian units. The central reason to denigrate a theory which refers to instrumental posits like centers of mass is precisely because we know that they lack causal powers. We see masses and the spring and the space in the middle where the center of mass would be located, were it a real object. We say, "There must be a better theory than the one which refers to centers of mass because there is nothing there." It is perilously easy for the indispensabilist to anoint as best those theories which avoid quantification over objects, like centers of mass, with no causal powers.

Retreat to an ideal theory thus allows one to import the same kinds of pre-theoretic prejudices against which the Quinean brandishes QI. Still, there are limits to the extent to which the indispensabilist can import such preferences, as QI itself shows. The defender of QI is a reluctant platonist, accepting mathematical objects only because there is no reasonably attractive alternative; most philosophers of mathematics agree that nominalist versions of standard scientific theories are less attractive than their counterparts which contain mathematical axioms.[24] But the defender of QI who appeals to an ideal theory to reject counter-examples like those of Azzouni and Maddy seems to engage in her/his own sort of double-talk.

So the double-talk criticism of EP is strong but not decisive. Like Colyvan's examples, it has the right form: a non-question-begging methodological principle. But its invocation is not categorical or unambiguous. Further, eleatics like Melia and Leng defend their double-talk by insisting that their nominalism is principled rather than arbitrary. The debate remains lively.

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[23] See Baker 2001: §2,3.

[24] See, for example, the debate between the indispensabilist Colyvan and the eleatic Melia in Melia 2000, Colyvan 2002, and Melia 2002. While they disagree on much, they agree on the attractiveness of the standard versions of scientific theories which include mathematics.

My central goal for the foregoing portions of this paper has been to establish that EP is not out of the running on the basis of Colyvan's examples. I do not mean to have defended EP. Perhaps the indispensabilist can argue convincingly that there are non-question-begging reasons to retreat to an ideal theory. Moreover, in §3, I put aside Oddie's worries about specifying the eleatic principle precisely; important distinctions often elude specification. But worries about causation, and the compatibility of differing formulations, can not be put aside forever. In addition, to establish EP, one would need to be more specific about the nature of, and criteria for, a best theory. We need better guidelines for balancing the various factors which determine the attractiveness of a theory and applying them in particular cases. That task is beyond the range of this paper.

§7: Beyond the Eleatic and the Indispensabilist

Despite efforts on both sides, the stalemate between the eleatic and the indispensabilist persists.[25] Eleatics claim, variously but contentiously, that mathematical terms are meaningless, that mathematical sentences are false (or merely vacuously true, in the cases of conditional or universal mathematical claims), or that mathematical objects do not exist. Indispensabilists claim, implausibly, that considerations arising from the construction of empirical theories should force us to believe in abstract objects which have no casual connection to ordinary objects. I end by sketching a view which I call autonomy platonism and which I believe is more plausible than either nominalism based on EP or platonism based on QI.

Autonomy platonism captures intuitions at the root of both the debate about the existence of mathematical objects and the methodological debate between EP and QI. The eleatic focuses on the

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[25] "Perhaps we reach a stand off here. Azzouni takes it that it is possible to get all the benefits of the mathematical explanations I offer from the notation alone. I am inclined to take the scientific explanations in question literally and see them as cases of mathematics (not just mathematical notation) doing the heavy lifting in the cases in question" (Colyvan 2012: 1033).

causal isolation of mathematical objects. The indispensabilist emphasizes the fact that our best scientific theories refer to mathematical and physical objects indiscriminately, without anointing some references as real and others as merely instrumental. In the remainder of this paper, I'll show how autonomy platonism embraces these aspects of both positions.

Autonomy platonism consists of an ontological claim and an epistemological claim. The ontological claim is that many mathematical propositions are true and refer to existing mathematical objects. The epistemological claim is that the justification of our mathematical beliefs does not depend on the uses of mathematics in empirical science. So '7+5=12', 'a tangent to a sphere in Euclidean space intersects the radius of that sphere at a right angle', and 'the power set of a set has a cardinality strictly larger than that of the original set' are all true sentences, referring to mathematical objects including numbers, shapes, and sets. Further, they are true regardless of any contingent features of the physical world, no matter what our scientific theories say or how they say it.

Autonomy platonism is thus a traditional, common-sense view. Like the indispensabilist, the autonomy platonist believes that mathematical objects exist. But the indispensabilist sees mathematical theories as dependent for their truth on their uses in formulations of empirical theories. The autonomy platonist rejects QI and appeals to independent reasons for believing that mathematical objects exist.

There are a variety of forms of autonomy platonism, and so a variety of kinds of independent reasons for believing in the existence of mathematical objects.[26] Some versions invoke a cognitive capacity which one could call mathematical intuition. While some philosophers find mathematical intuition contentious and this is not the place to defend it, such a capacity is, I believe, no more mysterious than ordinary *a priori* reasoning. Katz calls intuition, "A non-inferential, purely rational

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[26] Among philosophers who have held some version of autonomy platonism historically are Descartes, Hume, Frege, and Gödel. More recently, Jerrold Katz, John Burgess, Mark Balaguer, and Mark McEvoy have all explored versions of autonomy platonism; see Katz 1998, Burgess 1983, Balaguer 1998 (Chapters 2-4), and McEvoy 2004.

apprehension of the structure of an abstract object, that is, an apprehension that involves absolutely no connection to anything concrete" (Katz 1998: 44). McEvoy characterizes it as, "The ability to access, examine, and compare basic mathematical concepts" (McEvoy 2004: 434).

Still not all autonomy platonists appeal to intuition and it is not an essential component of autonomy platonism. Balaguer's plenitudinous platonism (FBP), for example, grounds our knowledge of mathematics in our basic ability to recognize consistency and entailment. On Balaguer's FBP, all consistent mathematical theories truly describe some mathematical domain. We can know about mathematical objects merely because we can recognize the consistency or inconsistency of mathematical theories, without appealing to the applications of mathematics in science or to mathematical intuition. Most philosophers agree that an ability to recognize consistency is not contentious since it is central to any account of our knowledge of logic. Even Hartry Field's fictionalism appeals to a natural ability to recognize consistency in defense of his dispensabilist construction for mathematics.[27]

Returning to the subject at hand, how autonomy platonism can capture most of the important intuitions supporting both the eleatic and the indispensabilist, I make two claims. First, the eleatic is correct about which existence claims we can infer from scientific theories. Second, the indispensabilist is correct about the existence of mathematical objects. Let's start with the first claim.

EP is eminently defensible when applied directly within empirical science. It allows us to rid ourselves of centers of mass and infinitely deep water waves, as well as caloric and ghosts, without presuming that there is some idealized version of science that avoids quantifying over them. EP can also explain why the indispensabilist retreats to the promise of an ideal theory in such cases. Given the failure of Colyvan's examples to find non-causal, non-mathematical quantifications within empirical science, there is no reason for the indispensabilist to reject EP within science. Even if the

[27] In Field 1980, Field invokes an object-level consistency operator to account for metalogical reasoning.

indispensabilist rejects using EP as a categorical criterion for determining what to believe exists, it may remain a useful rule of thumb within science. There is no real disagreement between the eleatic and the indispensabilist in non-mathematical cases; neither believes in infinitely deep water waves or centers of mass. Only the scientific anti-realist attacks EP within science, a view we can set aside here.

Like the indispensabilist, the autonomy platonist can emphasize the utility of EP within empirical science while limiting its scope to such theories. EP can do its work on centers of mass and ghosts while refusing to extend EP to formal theories like mathematics. The argument from the effectiveness of EP in identifying our commitments in empirical science to the denial of the existence of mathematical objects ignores the question of the independent legitimacy of mathematics. There is no incompatibility between EP, as a criterion for determining the commitments of scientific theory, and mathematical realism, which posits an independent realm of abstract objects. The eleatic principle can tell us how to read the claims of empirical science. We can appeal to independent reasons, like the consistency, stability, and cogency of mathematical theories, to accept mathematical axioms and to believe that mathematical objects exist.

My second claim about the attractiveness of autonomy platonism is that it captures the indispensabilist's claim that mathematical objects exist while denying its implausible allegation that our mathematical beliefs are justified just like our empirical beliefs. For the indispensabilist, all posits, mathematical and empirical, are on a par. For Quine, even ordinary objects like trees are posits in exactly the same way that sets are. But the roles of mathematical objects in our theories are different than the roles of other kinds of posits. While it is a simple logical fact that we can hold or cede any claim in a theory by making appropriate adjustments elsewhere, we actually hold mathematical theorems in the background when testing or confirming empirical claims. We do not allow them to be refuted in the ways in which we hold other claims open to disconfirmation.[28]

[28] Compare: "If the mathematical statements *M* are part of every competing hypothesis, then, no matter which hypothesis comes out best in the light of the observations, *M* will be part of that best hypothesis. *M* is not tested by this exercise, but is simply a background assumption common to the

Defenders of EP and QI agree that uses of mathematics in scientific theories are indispensable in practice. Their disagreement is over whether those uses are ontologically significant. The autonomy platonist accepts, with the indispensabilist, that they are, while also agreeing with the eleatic that the applications of mathematics within empirical theories are not reasons to believe that they are.

The defender of QI looks at a standard scientific theory, with its references to physical objects and mathematical objects, and sees no principled way of distinguishing them. Let's imagine that the theory contains Coulomb's law, CL.

CL $F = k\,|q_1q_2|/\,r^2$, where the electrostatic constant $k \approx 9 \times 10^9\ Nm^2/c^2$

First-order regimentations of CL quantify over both particles (with their charges) and real numbers (the electrostatic constant). The indispensabilist's claims that to isolate the mathematical elements, to call them merely instrumental, is arbitrary. The truth conditions for the mathematical axioms of our theories are of the same sort as the truth conditions for the other theorems, whether they include mathematical references or not. All theorems will be true if and only if the objects to which they refer exist and have the properties or relations ascribed to them. The autonomy platonist also captures this intuition while agreeing with the eleatic that such uses of mathematics are not reasons to believe in mathematical objects.

Mathematical theories are our most secure. Beliefs about mathematics are held more firmly and widely than even our best scientific theories. Where scientific theories are supplanted and ceded, replaced by better ones, mathematical theories are supplemented and extended. Euclidean geometry persists despite the development of non-Euclidean geometries and its replacement as the framework geometry for our best physics. Proofs from thousands of years ago remain paradigmatically good

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hypotheses under test" (Sober 1993: 45). See also Sober 1999 and Sober 2005.

mathematics. The core idea of QI is to defend a natural, literal, and sincere reading of our scientific theories with no double-talk. The autonomy platonist extends this natural and literal reading to mathematical theories themselves.

Any version of autonomy platonism thus captures core intuitions of both EP (except for its rejection of mathematical objects) and QI (except for the inference from the applications of mathematical theories to their truth). The autonomy platonist does reject the indispensabilist's claim that our uses of mathematics in science provide reasons to believe in mathematical objects. But the autonomy platonist agrees that our references to mathematical objects within scientific theories are robust. So both the indispensabilist and the autonomy platonist avoid the double-talk of quantifying over mathematical objects while not believing that they really exist. If we take mathematical objects to exist, the autonomy platonist can not be accused, as the eleatic may be, of ostrich nominalism regarding the mathematical elements of our theories.

Autonomy platonism based either on non-empirical (i.e. formal or *a priori*) evidence or on a brute knowledge of consistency is preferable both to nominalism based on EP and to indispensability platonism based on QI. A philosopher who is not independently committed to the non-existence of mathematical objects and who finds the relevant motivations for both EP and QI compelling should adopt this third view. Of course, more remains to be said, both in favor of autonomy platonism and against QI. Still, the arguments for autonomy platonism are independent of the goal of the first portion of this paper which is to show that Colyvan's attempts to break the stalemate between the defender of EP and the defender of QI leaves the debate between the eleatic and the indispensabilist right where it is.

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